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MEMOIRS, &c.

PART II.

ASTRONOMICAL AND MATHEMATICAL PAPERS.

New Method of Working a LUNAR OBSERVATION,
by NATHANIEL BOWDITCH, A.A.S.

VARIOUS methods have been proposed for calculating the correct distance of the sun or star from the moon, their apparent distance and altitudes being given : The easiest manner is by Shepard's Parallatic Tables ; but, as they cannot easily be procured, it is more common to calculate them by Lyon's, Witchel's, Danthorne's, or Croswell's methods. The chief advantage of the two last is, that there is no difference of cases ; but the logarithms must be obtained to great exactness, to six places of figures, by proportioning for the odd seconds. Lyon's and Witchel's methods do not labour under these disadvantages, as the logarithms need be taken to only four places of figures, and it is sufficiently exact to take them to the nearest minute ; but there are so many different ways of applying the corrections, particularly in Lyon's method, that learners, in general, find the rules very difficult. To remedy which, the following rule is proposed, which, in a degree, unites the advantages of the former, without labouring under their disadvantages ;

tages; it not being necessary to take the logarithms to more than four places of figures, and to the nearest minute; and there is no difference of cases, the corrections being always applied in the same manner, whatever may be the distance and altitudes.

R U L E.

From the proportional logarithm of the moon's horizontal parallax, (found in Tab. XIII. Maskelyne's Requisite Tables, or Tab. IX. of tenth edition of Hamilton Moore's Epitome) its index being increased by 10, subtract the log. sine of the moon's zenith distance, and the remainder is the prop. log. of her parallax in altitude; from which, subtracting her refraction, the remainder will be the correction of the moon's altitude.

From the sun's refraction (Tab. I. Req. Tab.) take his parallax (Tab. III. ib.) the remainder will be the correction of the sun's altitude.

The star's refraction (Tab. I. Req. Tab.) is its correction in altitude.

Add together the sun's zenith distance, the moon's zenith distance, and the apparent distance of the sun and moon, and note the half sum of these three numbers.

To the constant logarithm 9.6990 add the cosecant of the half sum and the sine of the apparent distance; the sum, rejecting 20 in the index, will be a reserved logarithm.

To this reserved logarithm, add the sine of the sun's zenith distance, the cosecant of the difference between the moon's zenith distance and the half sum, and the prop. log. of the correction of the sun's altitude ; the sum, rejecting 30 in the index, will be the prop. log. of the first correction.

To the reserved logarithm add the sine of the moon's zenith distance, the cosecant of the difference between the star's zenith distance and the half sum, and the prop. log. of the correction of the moon's altitude ; the sum, rejecting 30 in the index, will be the prop. log. of the second correction.

Then to the apparent distance, add the correction of the moon's altitude, and the first correction, and subtract the sum of the second correction, and the correction of the sun's altitude ; the remainder will be the moon's distance corrected.

Take the difference between the correction of the moon's altitude and the second correction, and with that, enter in the side column of Tab. XIII. (of Req. Tab. or Tab. X. Ham. Moore) and under the corrected distance, take out the corresponding number of seconds, which reserve. Under the same distance, and opposite the correction of the moon's altitude, take out the number of seconds ; the difference between this and the number of seconds above reserved, must be added to the corrected distance, when less than 90 degrees, but subtracted when greater than 90 ; the sum or difference will be the true distance.

The

ASTRONOMICAL AND

The rule is the same for a star and the moon, as for the sun and moon, only in the rule, the word star is to be read instead of sun.

EXAMPLE.

Given apparent central distance of moon and star $34^{\circ} 25' 42''$,
moon's zenith distance $27^{\circ} 18'$, star's zenith distance $60^{\circ} 14'$,
prop. log. of moon's horizontal parallax 4741 : Required
the corrected distance.

Prop. log. moon's hor. par.	10.4741	* Zenith distance,	60 14
Sine $\searrow$ zen. dist.	<u>9.6613</u>	$\searrow$ Zenith distance,	27 18
		Apparent distance,	<u>34 26</u>
Prop. log. $\searrow$ corr. in par. $27' 42''$	8126		
$\searrow$ Refraction,	<u>0 29</u>		Sum) 121 58
Corr. $\searrow$ altitude,	<u>27 13</u>		$\frac{1}{2}$ Sum 60 59
* Refraction or correction,	<u>1' 39''</u>	$\frac{1}{2}$ Sum — $\searrow$ zenith distance,	<u>33 41</u>
		$\frac{1}{2}$ Sum — * zenith distance,	<u>0 45</u>
Constant logarithm,	9.6990		
$\frac{1}{2}$ Sum, 60 59 Cosecant,	10.0583		
Appar. dist. 34 26 Sine,	<u>9.7524</u>		
Reserved logarithm	9.5097		
* Zen. dist. 60 14 Sine,	9.9385	$\searrow$ Zen. dist. 27 18 Sine,	9.6615
$\frac{1}{2}$ Sum — $\searrow$ zen. dist. 33 41 Cosec.	10.2560	$\frac{1}{2}$ Sum — * zen. dist. 0 45 Cosec.	11.8830
Correct. * alt. 1' 39'' Pro. log.	<u>2.0378</u>	Correct. $\searrow$ alt. 27' 13'' Pro. log.	<u>0.8204</u>
1st Correction, 3' 16'' Pro. log.	<u>1.7420</u>	2d Correction, 2' 24'' Pro. log.	<u>1.8746</u>
Apparent distance,	34 25 42		
Add { Correct. $\searrow$ alt.	27 13		
1st Correction,	<u>3 16</u>		
	34 56 11		
Subtract { Correct. * alt. 1' 39'' }	4 3		
2d Correction, 2' 24'' }	<u>34 52 8</u>		
Add correction from Tab. XIII.	<u>1</u>		
Correct distance,	34 52 9		

The foregoing directions will give the correct distance sufficiently near for any nautical purpose whatever ; but, if the distance were wanted correct to the nearest second, allowance ought to be made for the different temperature of the air, denoted by the Thermometer and Barometer, as it affects the refraction. A fourth correction ought also to be applied, which may be found as follows.

Take the difference between the first correction and the correction of the sun's altitude, which call S. and the difference between the second correction and the correction of the moon's altitude which call M. then add together, (1.) Half the sum of the prop. logs. of the sum and difference of the correction of the sun's altitude and correction S. (2.) Half the sum of the prop. logs. of the sum and difference of the correction of the moon's altitude and correction M. (3.) The log. sine of the corrected distance. (4.) And the constant logarithm 9.5029. The sum of these four logs. rejecting 20 in the index, will be the prop. log. of this correction, expressed in seconds and thirds ; which, being added to the corrected distance, gives the true distance ; but this correction never amounts to more than a few seconds, and is always neglected.

Demonstration of the Rule given for working a Lunar Observation.

LET Z (Plate I. fig. 1.) be the zenith, ZsS , ZMm be two vertical circles, in which let S be the true, s the apparent place of the sun, and M the true, m the apparent place of the moon. Zs and Zm the apparent, and ZS , ZM the true zenith distances,

tances, $S M$ and $s m$ the true and apparent distances respectively. From S and M on $m s$ let fall the perpendiculars $S s$, $M R$ forming the right angled triangles $S s r$, $M R m$, in which the angles at s and m are respectively equal to the angles $Z s m$, $Z m s$, then $Rr = ms \mp sr \pm Rm$, the upper sign being used for the corrections $s r$, $R m$, or the lower, according as the angles at s and m are respectively greater or less than 90 degrees. It is in finding substitutes for these corrections $s r$, $R m$ that our method differs from others; for after the value of Rr is obtained, from Tab. XIII. of *Req. Tables*, as in Lyon's method, we find the difference between the arches $r R$ and $r M$, which being applied, gives $r M$; afterwards, if you think proper, you may apply our fourth correction, which is equal to the difference between $M r$ and $M S$, and is the same as the fourth correction of Lyon's; and you will have the true distance $M S$. These two last corrections, as well as the rule for finding $S s$ and $M m$, being the same, as in Lyon's or Witchel's method, require no new demonstration, but the other part, depending on the corrections $s r$, $R m$ may be demonstrated in the following manner:

On the center C describe a semicircle $B D A$, (Plate I. fig. 2.) on which take the arc $B D =$ angle at s . Join $A D$, and from C on it let fall the perpendicular $C F$, then from the similar triangles $A F C$, $A E D$ we have $A F (= \frac{1}{2} A D = \text{cofine } \frac{1}{2} \text{ arch } B D) : A C (1) :: A E (= 1 \mp \text{cofine } B D) : A D (= 2 \text{ cofine } \frac{1}{2} \text{ arch } B D)$ in which, as above, the upper sign is to be used, when $B D$ or the angle s is more than 90 degrees, and

and the lower when less. Therefore,

$1 \mp \text{cofine angle at } s = \frac{2}{\text{cof. } \frac{1}{2} \text{ angle at } s} = \frac{S s \mp S r}{2 S s}$
 (because of the right angled triangle $S r s$) and in logarithms, $2 \times \log. \text{cofine } \frac{1}{2} \text{ angle at } s = \log. S s \mp s r - \log. S s$
 $\div \log. 2$: but by common spherics $2 \log. \text{cofine } \frac{1}{2} \text{ angle } s$
 $= \log. \text{cofecant } Z s + \log. \text{cofecant } s m + \log. \text{fine } \frac{1}{2} \text{ fum three}$
 $\text{fides } Z s, s m, Z m, + \log. \text{fine } \frac{1}{2} \text{ fum three fides} - \text{fide } Z m,$
 which is therefore $= \log. S s \mp s r - \log. S s - \log. 2$; whence
 $\log. S s \mp s r, = \log. S s + \log. 2 + \log. \text{cofec. } Z s + \log.$
 $\text{cofec. } s m + \log. \text{fine } \frac{1}{2} \text{ fum three fides} + \log. \text{fine } \frac{1}{2} \text{ fum} - Z m,$
 or by using proportional logarithms, prop. log. $S s \mp s r = \text{pr.}$
 $\log. S s - \log. 2$ (or $+ 9.6990$) $- \log. \text{cofec. } Z s$ (or $+ \log.$
 $\text{fine } Z s) - \log. \text{cofec. } s m$ (or $+ \log. \text{fine } s m) - \log. \text{fine } \frac{1}{2} \text{ fum}$
 or $(+ \log. \text{cofec. } \frac{1}{2} \text{ fum}) - \log. \text{fine } \frac{1}{2} \text{ fum} - Z m$ (or $+ \log.$
 $\text{cofec. } \frac{1}{2} \text{ fum} - Z m$) but our reserved log. is $= 9.6990 +$
 $\text{cofec. } \frac{1}{2} \text{ fum} + \text{fine } s m$, therefore $S s \mp s r - \text{reserved log.} +$
 $\log. \text{cofec. } Z s + \log. \text{cofec. } \frac{1}{2} \text{ fum} - Z m + \text{prop. log. } S s,$
 but these numbers are exactly the same as those made use of
 for finding our first correction; therefore $S s \mp s r = 1^{\text{st}}$ cor-
 rection. In the same manner it may be proved, that $M m \mp$
 $m R = 2^{\text{d}}$ correction. Consequently, $m s + \text{correction} \gg \text{alt.}$
 $(M m) + 1^{\text{st}}$ correction (or $+ S s \mp s r$) $- \text{cor. } \odot \text{ alt. (or } -$
 $S s) - 2^{\text{d}}$ correction (or $- M m \pm m R) = m s \mp s r \pm m R =$
 $R r$ as was shewn on last page, which proves that our 1st and
 2^d corrections, applied according to our rules, will give the
 value of $R r$ required.

Appendix to the Method of working a Lunar Observation.

TO simplify the calculations of a Lunar Observation, some small additions may be made in the tables of Logarithmic sines, which will considerably facilitate the work,

In the first place, when regulating a timepiece, by means of an altitude of the Sun or other object, the common rule for finding the apparent time gives the cosine of half the hour angle; from which the angle itself is found and doubled, and generally (by seamen) turned into time, by means of a table calculated for that purpose. All this may be avoided by putting the corresponding time in the side columns of the table of sines (the margin of these tables being generally wide enough for that purpose.) Adjoined is an example of the table for 21 degrees: at the top they are all marked P. M. at the bottom, A. M. so that if the log. cosine was 9.96991, by inspection, we should immediately find, that the time in the side column corresponding, was 2^h 48' 40" P. M. which is the time from passing the meridian of the object. If the logarithm was 9.55597, the cosine would be at bottom, and the time corresponding 2^h 48' 40", is to be subtracted from 12^h to obtain the time from passing the meridian of the object. These tables may be thus marked in three or four hours, nothing can be more simple than the method of doing it, as it requires only a constant addition of 8" for every minute of the circle.

Again, in all the tables of sines, which I have seen, the degrees are only marked from 0 to 90: Now, it often occurs in
finding

finding the apparent time, sun's azimuth, &c. that the degrees, whose logarithms are wanted, exceed 90; in which case, the common method is to subtract it from 180, and make use of the remainder. To

avoid this subtraction, the degrees above 90 to 134 may be marked at the left hand column of the bottom of the table; and from 135 to 179 at the right hand column of the top of the table; the numbers to be marked down, being the difference between the printed numbers and 179: as in the example, 158 is the difference between 179 and 21; and 111 is the difference between 179 and 68, the odd minutes must be found in the column directly above or below where

2 Hours P.M.		21 Degrees.					158
h	m	Sine.	Cofine.	Tang.	Cotan.	Secant.	Cofec.
2	48	0					60
	8	1					59
	16	2					58
	24	3					57
	32	4					56
	40	5	9.55597	9.96991			55
	48	6					54
	56	7					53
49	4	8					52
	12	9					51
	20	10					50
	28	11					49
	36	12					48
	44	13					47
	52	14					46
50	0	15					45
	8	16					44
	16	17					43
	24	18					42
	32	19					41
	40	20					40
&c. &c.	&c.						&c.
54	0	45					15
	8	46					14
	16	47					13
	24	48					12
	32	49					11
	40	50					10
	48	51					9
	56	52					8
55	4	53					7
	12	54					6
	20	55					5
	28	56					4
	36	57					3
	44	58					2
	52	59					1
2	56	0					C
		Cofine.	Sine.	Cotan.	Tang.	Cofec.	Secant.

2 Hours A.M. 111 68 Degrees.

the number of degrees are marked; thus the sine of $111^{\circ} 5'$ is

C

9.96991,

9.96991, and the sine of $158^{\circ} 55'$ is 9.55597. These numbers may be marked through the whole table in 15 or 20 minutes. I have marked some requisite tables in this manner, and find it simplifies the calculations to young calculators.

In general three observers are required to make the requisite observations for determining the longitude; one to measure the distance of the two bodies, the other two to take their altitudes. If the altitudes were not observed, it has been customary to calculate them, by means of the given time of observation, the latitude of the place, declinations and right ascensions of the objects, which are found from the nautical almanacs. This calculation for the stars and moon is lengthy, and is also liable to some error, on account of the uncertainty of the ship's longitude, which will alter the numbers taken from the ephemeris. The following method I have made use of, for a long time, to simplify the work, and I have lately observed that Mr. Vince, in his new system of astronomy proposes the same method.

Observe the altitudes of the objects before and after you have measured their distance, and note the times of observation (for greater exactness a number of distances may be measured and their corresponding times noted, taking the mean of all the distances and times for the true measured distance and time.) Then add together the prop. log. of the variation of altitude, between the two observations and the prop. log. of the time elapsed between observing the first altitude and measuring the mean distance; from the sum subtract the proportional log. of the time elapsed, between the observation of the altitudes, the

remainder will be the proportional log. of the variation in altitude, which is to be added to the first observed altitude, if increasing, or subtractive if decreasing. The sum or difference will be the required altitude at the time of observation, which is to be corrected, as usual, for the sun's semidiameter, and dip of the horizon. In this manner I have often obtained the altitudes, in much less time than I could have obtained them by other calculations, and sufficiently exact for any nautical purpose whatever.

EXAMPLE.

Suppose the times, altitudes, distances, &c. to have been observed, as below, it is required to find the altitudes at the time of measuring the mean distance.

Time.	Dist. obf. $\odot$ $\nearrow$ n. l.	Time.	Obf. alt. $\nearrow$ l. l.	Time.	$\odot$ Obf. alt. l. l.
$2^h 3' 20''$	$40^\circ 0' 0''$	$2 2 0$	$20 46$	$2 2 30$	$40 20$
$4 20$	$0 30$	$2 6 10$	$21 20$	$2 7 0$	$39 12$
$5 50$	$1 30$				
Mean, $2 4 30$	$40^\circ 0' 40''$	Dif. $0^h 4 10$	$0^\circ 34$	Dif. $4 30$	$1 8$
	Dif. alt. $0^\circ 34'$ prop. log. 0.7238		Dif. alt. $1^\circ 8'$ prop. log. 0.4228		
Mean time, $2 4 30$	} Dif. $2' 30''$ p. log. 1.8573	Mean time, $2 4 30$	} Dif. $2' 0''$ p. log. 1.9542		
Time of 1st obf. $2 2 0$		Time of 1st alt. $2 2 30$			
	2.5811		2.3770		
	Subtract prop. log. $4' 10''$ 1.6355		Subtract log. $4' 30''$ 1.6021		
	9456				
Corr. of alt.	$20'$	Corr. of altitude,	$0^\circ 30'$ p. log. 7749		
Add to 1st alt.	$20 46$	To be subtracted from the	} $40 20$		
		1st observed altitude.			
Alt. $\nearrow$ at time of obf.	$21 6$	Sun's alt. at time of obf.	$39 50$		

So that at the time $2^h 4' 30''$, when the moon's mean observed distance was $40^\circ 0' 46''$, the $\nearrow$ alt. l. l. was $21^\circ 6'$, and the sun's observed altitude l. l. $39^\circ 50'$, after which the calculation may be made as usual.

NOTE. Since this paper was communicated, Mr. Bowditch has compiled and published an Epitome of Navigation, entitled, "*The New Practical Navigator*," into which his new method of working a Lunar Observation is introduced. In that work, to which the reader is referred, will be found a substitute for the Tables mentioned in the above communication.